

Quantum-Statistical Kinetic Equations

D. Loss¹ and H. Schoeller¹

Received November 7, 1988

Considering a homogeneous normal quantum fluid consisting of identical interacting fermions or bosons, we derive an exact quantum-statistical generalized kinetic equation with a collision operator given as explicit cluster series where exchange effects are included through renormalized Liouville operators. This new result is obtained by applying a recently developed superoperator formalism (Liouville operators, cluster expansions, symmetrized projectors, P_q -rule, etc.) to nonequilibrium systems described by a density operator $\rho(t)$ which obeys the von Neumann equation. By means of this formalism a factorization theorem is proven (being essential for obtaining closed equations), and partial resummations (leading to renormalized quantities) are performed. As an illustrative application, the quantum-statistical versions (including exchange effects due to Fermi-Dirac or Bose-Einstein statistics) of the homogeneous Boltzmann (binary collisions) and Choh-Uhlenbeck (triple collisions) equations are derived.

KEY WORDS: Kinetic equations; exchange effects; renormalized cluster series; quantum-statistical Boltzmann and Choh-Uhlenbeck equations.

1. INTRODUCTION

Nonequilibrium cluster expansion techniques have been a useful tool for the microscopic analysis of the N -body dynamics of classical fluids. Based on these techniques, generalized kinetic equations can be derived from which, for instance, the Boltzmann equation and its first correction, the Choh-Uhlenbeck equation, are obtained in a systematic approximation procedure (see, e.g., refs. 1-4). Other examples, successfully studied in kinetic theory by means of cluster expansions, are the well-known long-time tails of equilibrium time correlation functions,⁽⁴⁻⁶⁾ the nonanalytic density dependence of transport coefficients,^(7,8) the divergence of Burnett coefficients,^(9,10) etc. (see also ref. 11, especially for a bibliography).

¹ Institut für Theoretische Physik der Universität Zürich, CH-8001 Zürich, Switzerland.

Motivated by the progress made in this area of statistical mechanics we have started, in ref. 12 (hereafter referred to as I), the development of a new formalism directed at generalizing the methods of classical kinetic theory, in particular nonequilibrium cluster expansion techniques, to normal quantum fluids. (For other studies with a similar objective see, e.g., refs. 13–19.) There, we were concerned with the microscopic evaluation of the momentum autocorrelation function in semiclassical systems of N identical particles interacting pairwise via a translationally invariant short-range repulsive potential (with no bound states). By semiclassical we mean here that the system is described quantum mechanically, whereas the particles obey classical Boltzmann statistics. It turned out that the autocorrelation function has the same long-time tail in this case as in classical hard-sphere systems.^(5,20) This analysis was based on the superoperator formalism, in particular on Liouville operators and projectors, and on a new concept, the P_q -rule and P_q -singularity (the latter being a generalization of van Hove's diagonal singularity^(21,22)), by the help of which the various terms occurring in the nonequilibrium cluster expansion could be discussed systematically in the thermodynamic and long-time limit.

This investigation, however, was restricted to the semiclassical regime and does therefore not apply to low-temperature systems, because in this case exchange (degeneracy) effects due to Fermi–Dirac (FD) or Bose–Einstein (BE) statistics are not negligible.

In a subsequent work⁽²³⁾ (hereafter referred to as II), also concerned with the evaluation of equilibrium time correlation functions, we extended our semiclassical formalism to the quantum-statistical case. The main result in this investigation is a general formula [see (II.3.44)] from which, as a first application, the well-known⁽¹⁷⁾ quantum-statistical Boltzmann equation value of time-integrated correlation functions (with exchange-modified scattering cross section) is obtained microscopically. This general formula may then also serve, e.g., as a suitable starting point for the determination of the long-time tails of correlation functions at low temperatures. This, however, will not be done here, although the renormalized cluster series found in the present work might be of great use in such an investigation.

The objective of the present paper is to derive a new exact generalized kinetic equation for homogeneous (i.e., spatially uniform) nonequilibrium quantum fluids (such as, e.g., normal ^3He or ^4He) consisting of N interacting fermions or bosons. By a straightforward and systematic approximation procedure, we then obtain here the quantum-statistical version of the homogeneous nonlinear Boltzmann equation (with exchange modified, time-dependent scattering cross section) in the binary collision approximation. This known result was first derived by Boercker and Dufty⁽¹⁶⁾ in a semiphenomenological approach based on the quantum BBGKY

hierarchy. Moreover, in the triple collision approximation we obtain the quantum-statistical version of the classical⁽²⁴⁾ or semiclassical⁽²⁵⁻²⁷⁾ Choh-Uhlenbeck equation, a result which we believe to be new.

For a first orientation without any technical details let us now sketch the main line we follow in deriving the above-mentioned generalized kinetic equation. By kinetic equation we mean, as usual, a closed nonlinear integrodifferential equation describing the time evolution of the one-particle reduced distribution operator $\rho_1(t)$. The starting point of our discussion is the von Neumann equation for the density matrix $\rho(t)$ describing the time evolution of the nonequilibrium system under consideration. Making use of the technical tools (superoperators, symmetrized projectors, P_q -rule, etc.) introduced and discussed in I and II, and performing cluster expansions, we readily obtain the time derivative of the one-particle reduced distribution operator $\dot{\rho}_1(t)$ expressed as some functional of the diagonal part (in the momentum eigenstates) of the s -particle reduced distribution operators $P^{1\dots s}\rho_{1\dots s}(t)$. Now, under certain conditions on the initial distribution a factorization theorem will be proven, which says that this diagonal part of $\rho_{1\dots s}(t)$ factorizes exactly in the thermodynamic limit and can be replaced by the product $\rho_1(t)\cdots\rho_s(t)$. Thereby we obtain a closed equation for $\rho_1(t)$ where only one-particle distribution operators $\rho_i(t)$ occur. In passing, we note that analogous forms of this factorization theorem were derived and used in the evaluation of equilibrium time correlation functions in I and II. In classical or semiclassical systems the thus obtained equation would already represent the desired result. However, in the quantum-statistical case considered here, exact partial resummations of the cluster series can be performed leading to a renormalized generalized kinetic equation [see Eq. (4.2)] where many-body effects due to FD or BE statistics are incorporated explicitly through renormalized Liouville operators being now linear functionals of $\rho_i(t)$. This exact equation, which is given as a cluster series of very similar structure as its (semi-) classical counterpart, represents then the main result of our general discussion, from which the quantum-statistical versions of the Boltzmann and Choh-Uhlenbeck equations are straightforwardly obtained in the Markovian limit.

The paper is organized in the following way. In Section 2, besides introducing some basic definitions, we establish the above-mentioned central factorization theorem. Thereby we make use of nonequilibrium cluster expansions and the P_q -rule discussed in I. In Section 3 we start the actual derivation of the generalized kinetic equation. We perform there a nonequilibrium cluster expansion of the Laplace transform of the dynamical factor $(\varepsilon + iL)^{-1}$ (L is the Liouville operator) and decompose the resulting cluster expressions into their diagonal and nondiagonal parts with the help

of symmetrized projectors. Then, inserting the factorization theorem, we arrive at a closed equation for $\rho_1(t)$. In Section 4 we perform partial resummations which lead to the exact renormalized generalized kinetic equation. In Section 5 we consider the binary collision and Markovian approximations of this equation which yield the quantum-statistical version of the homogeneous Boltzmann equation. In Section 6 we derive the quantum-statistical version of the Choh–Uhlenbeck equation by retaining binary and triple collision terms, including the first time retardation correction to the Markovian approximation. We also discuss there the semiclassical limit of this equation, thereby obtaining the semiclassical triple collision operator discussed by Résibois⁽²⁵⁾ within the Brussels formalism.⁽¹³⁾

2. FACTORIZATION THEOREM

In this section, introducing some basic definitions and concepts, we establish a factorization theorem for the reduced nonequilibrium density operator which will be essential for the derivation of the generalized kinetic equations discussed in the next sections. For an instructive heuristic discussion of the factorization problem in classical systems see, e.g., ref. 4, p. 216.

We consider a homogeneous quantum-statistical system of N identical fermions or bosons of mass m in a periodicity volume Ω , which interact via a short-range two-body potential with an arbitrarily strongly repulsive core and with no bound states. Denoting by \mathbf{p}_i the momentum operator of the i th particle, the Hamiltonian operator for this system is given as

$$H = H_0 + V = \sum_{i=1}^N \frac{\mathbf{p}_i^2}{2m} + \sum_{i < j}^N V_{ij} \quad (2.1)$$

with $V_{ij} = V(|\mathbf{x}_i - \mathbf{x}_j|)$. The quantity of central interest here is the normalized density operator $\rho(t)$, which specifies the statistical state of the nonequilibrium system at time t . Its time evolution is governed by the von Neumann equation (we set $\hbar = 1$)

$$\frac{\partial}{\partial t} \rho(t) = -iL\rho(t) \quad (2.2)$$

L is the Liouville operator,² defined by

$$LA = [H, A] \quad (2.3a)$$

² The Liouville operator belongs to the class of superoperators⁽⁴¹⁾ which are formally defined as linear operators acting on ordinary Hilbert-space operators.

where here and in the following A denotes an ordinary Hilbert-space operator. Analogously, one defines

$$L_0 A = [H_0, A] \tag{2.3b}$$

$$L_V A = [V, A] \tag{2.3c}$$

and

$$L_{ij} A = [V_{ij}, A] \tag{2.3d}$$

The solution of Eq. (2.2) reads

$$\rho(t) = e^{-iL(t-t_0)} \rho(t_0), \quad t \geq t_0 \tag{2.4}$$

The quantum-statistical expectation value of any observable A at time t is then given by

$$\langle A \rangle_t = \text{Tr } A \rho(t) \tag{2.5}$$

where the trace Tr is to be taken over a complete orthonormal set of (anti-) symmetrized states. They may be chosen as eigenstates of the total momentum and are of the form

$$\left(\frac{N!}{\prod_i n_i!} \right)^{1/2} \pi |\mathbf{k}_1 \cdots \mathbf{k}_N \rangle \tag{2.6}$$

Here,

$$|\mathbf{k}_1 \cdots \mathbf{k}_N \rangle = |\mathbf{k}_1 \rangle \times \cdots \times |\mathbf{k}_N \rangle \equiv |k \rangle \tag{2.7}$$

is the direct product of single-particle momentum eigenstates; π denotes the projecting operator which (anti-) symmetrizes the product states:

$$\pi = \frac{1}{N!} \pi^{1 \cdots N} = \frac{1}{N!} \sum_{\sigma \in S_N} \pi_\sigma \tag{2.8}$$

$$\pi_\sigma |k \rangle = \eta^{|\sigma|} |\mathbf{k}_{\sigma(1)} \cdots \mathbf{k}_{\sigma(N)} \rangle \tag{2.9}$$

$$|\sigma| = \begin{cases} 2 & \text{for even permutations} \\ 1 & \text{for odd permutations} \end{cases}$$

$$\eta = \begin{cases} 1 & \text{for bosons} \\ -1 & \text{for fermions} \end{cases}$$

The sum in (2.8) runs over all permutations σ of N indices. n_i indicates the number of identical \mathbf{k} -vectors of type i . Since the state (2.6) remains

unchanged (apart from the sign in the fermionic case) under a permutation of the vectors \mathbf{k}_i , only those states of the form (2.6) need to be considered in Eq. (2.5) whose sets $\{\mathbf{k}_1, \dots, \mathbf{k}_N\}$ are different. Therefore Eq. (2.5) reads

$$\begin{aligned} \langle A \rangle_t &= \sum_{\{\mathbf{k}_1, \dots, \mathbf{k}_N\}} \frac{N!}{\prod_i n_i!} \langle k | \pi A \rho(t) \pi | k \rangle \\ &= \sum_{\mathbf{k}_1, \dots, \mathbf{k}_N} \langle k | \pi A \rho(t) \pi | k \rangle \end{aligned} \quad (2.10)$$

Since the physical quantities A and $\rho(t)$ are symmetric operators, one has

$$\pi A \rho(t) \pi = A \rho(t) \pi \pi = A \rho(t) \pi \quad (2.11)$$

and hence

$$\langle A \rangle_t = \text{Tr}_{1 \dots N} A \rho(t) \pi \quad (2.12)$$

where $\text{Tr}_{1 \dots N} \equiv \text{Tr}_1 \dots \text{Tr}_N$ denotes the trace for Boltzmann statistics.

If A now consists of a sum of s -particle operators, i.e.,

$$A = \sum_{1 \leq i_1 < \dots < i_s \leq N} A_{i_1 \dots i_s} \quad (2.13)$$

it follows from Eq. (2.6) that

$$\langle A \rangle_t = \frac{1}{s!} \text{Tr}_{1 \dots s} A_{1 \dots s} \rho_{1 \dots s}(t) \quad (2.14)$$

where

$$\rho_{1 \dots s}(t) = \frac{N!}{(N-s)!} \text{Tr}_{s+1 \dots N} \rho(t) \pi \quad (2.15)$$

is the reduced s -particle density operator at time t . The symmetrization implicit in $\rho_{1 \dots s}(t)$ becomes more evident by noticing that

$$\rho_{1 \dots s}(t) = \frac{1}{s!} \pi^{1 \dots s} \rho_{1 \dots s}(t) \quad (2.16)$$

This relation follows immediately from the ‘‘cluster’’ representation

$$\pi = \frac{1}{N!} \pi^{1 \dots s} (1 + \pi_{1s+1} + \dots + \pi_{ss+1}) \dots (1 + \pi_{1N} + \dots + \pi_{N-1N}) \quad (2.17)$$

and $(\pi^{1 \dots s})^2 = s! \pi^{1 \dots s}$. Now, at the initial time $t = t_0$, $\rho_{1 \dots s}(t_0)$ is assumed to factorize (up to correlations due to FD or BE statistics) as follows:

$$\rho_{1 \dots s}(t_0) = \pi^{1 \dots s} \rho_1(t_0) \dots \rho_s(t_0) \quad (2.18)$$

Here,

$$\pi^{1\dots s} = (1 + \pi_{12})(1 + \pi_{13} + \pi_{23}) \cdots (1 + \pi_{1s} + \cdots + \pi_{s-1s})$$

takes care of the correct symmetry property of the initial statistical state of the fermion or boson system [note that Eq. (2.18) is consistent with Eq. (2.16)]. However, initial correlations due to particle interactions are neglected in this way. In the following we will consider such systems for which the factorization assumption (2.18) is possible.

This product form in Eq. (2.18), which will be used as initial condition in the derivation of generalized kinetic equations in the next sections, is in general not maintained for times $t > t_0$. However, for the *diagonal part* of $\rho_{1\dots s}(t)$ the following central factorization theorem holds true in the thermodynamic limit (i.e., $\Omega, N \rightarrow \infty$, with $N/\Omega = n$ finite):

$$P^{1\dots s} \rho_{1\dots s}(t) = P^1 \rho_1(t) \cdots P^s \rho_s(t) \quad \text{for all } t \geq t_0 \quad (2.19)$$

Here $P^{1\dots s} = P^1 \cdots P^s$ denotes the superoperator which projects on the diagonal part of an ordinary operator; explicitly,

$$\begin{aligned} (P^{1\dots s} \rho_{1\dots s}(t))_{k^s k'^s} &\equiv \langle k^s | P^{1\dots s} \rho_{1\dots s}(t) | k'^s \rangle \\ &= (\rho_{1\dots s}(t))_{k^s k'^s} \delta_{\mathbf{k}_1 \mathbf{k}'_1} \cdots \delta_{\mathbf{k}_s \mathbf{k}'_s} \end{aligned} \quad (2.20)$$

with $k^s = (\mathbf{k}_1, \dots, \mathbf{k}_s)$. The P^i on the rhs of Eq. (2.19) can actually be omitted here, since for the considered homogeneous system $P^i \rho_i(t) = \rho_i(t)$.

We note that in ref. 13, Chapter 19, a similar factorization theorem (proven⁽²⁸⁾ within the Brussels formalism) is discussed and used in deriving generalized kinetic equations. Furthermore, we note that in I and II [see Eqs. (I.3.58) and (II.3.42)] analogous forms of Eq. (2.19) have been derived in connection with the evaluation of equilibrium time correlation functions. The following proof is therefore analogous to those in I and II.

To begin with the proof of formula (2.19), we insert Eq. (2.4) into Eq. (2.15) and by using $(L\rho)\pi = L(\rho\pi)$ we obtain

$$P^{1\dots s} \rho_{1\dots s}(t) = \frac{N!}{(N-s)!} \text{Tr}_{s+1\dots N} P e^{-iL(t-t_0)} \rho(t_0) \pi \quad (2.21)$$

with $P = P^1 \cdots P^N$

Next we decompose $\exp[-iL(1 \cdots N)\tilde{t}]$ [$L \equiv L(1 \cdots N)$, $\tilde{t} \equiv t - t_0$] into the following cluster series, for all $N = 1, 2, \dots$:

$$\begin{aligned} e^{-iL(1)\tilde{t}} &= e^{-iL_0(1)\tilde{t}} = U(1) \\ e^{-iL(12)\tilde{t}} &= U(12) + U(1)U(2) \\ e^{-iL(123)\tilde{t}} &= U(123) + U(12)U(3) + U(13)U(2) \\ &\quad + U(23)U(1) + U(1)U(2)U(3) \end{aligned}$$

etc. For arbitrary N this may be written as

$$e^{-iLi} = \sum_{l=1}^N \sum_{I_1, \dots, I_l} U(I_1) \cdots U(I_l) \quad (2.22)$$

where $I_1 \cup \cdots \cup I_l = \{1, \dots, N\}$, $I_i \cap I_j = \emptyset$ for $i \neq j$, and $U(\emptyset) \equiv 1$. Inserting (2.22) into Eq. (2.21), we see that all clusters $U(i)$, $1 \leq i \leq N$, yield 1, since, with $PL_0 = 0$,

$$P^i U(i) = P^i \quad (2.23)$$

Let us now consider the following three typical cases which occur in Eq. (2.21) after insertion of (2.22):

$$\begin{aligned} & \frac{N!}{(N-s)!} (N-s)(N-s-1) \text{Tr}_{s+1 \dots N} P U(s+1, s+2) \rho(t_0) \pi \\ &= \text{Tr}_{s+1, s+2} P^{1 \dots s+2} U(s+1, s+2) \rho_{1 \dots s+2}(t_0) \end{aligned} \quad (2.24)$$

$$\begin{aligned} & \frac{N!}{(N-s)!} (N-s)(N-s-1) \text{Tr}_{s+1 \dots N} P U(1, s+1, s+2) \rho(t_0) \pi \\ &= \text{Tr}_{s+1, s+2} P^{1 \dots s+2} U(1, s+1, s+2) \rho_{1 \dots s+2}(t_0) \end{aligned} \quad (2.25)$$

and

$$\begin{aligned} & \frac{N!}{(N-s)!} (N-s)(N-s-1) \text{Tr}_{s+1 \dots N} P U(1, 2, s+1, s+2) \rho(t_0) \pi \\ &= \text{Tr}_{s+1, s+2} P^{1 \dots s+2} U(1, 2, s+1, s+2) \rho_{1 \dots s+2}(t_0) \end{aligned} \quad (2.26)$$

The first case yields zero, since

$$\text{Tr}_I U(I) A = 0 \quad (2.27)$$

as can be seen by expressing $U(I)$ in terms of $\exp[-iL(I')i]$ [invert the cluster expansion (2.22)] and then by using the fact that

$$\text{Tr}_I e^{-iL(I')i} A = \text{Tr}_I e^{-iH(I')i} A e^{iH(I')i} = \text{Tr}_I A \quad (2.28)$$

Therefore, the clusters U have to contain at least one index from the set $\{1, \dots, s\}$. However, if they contain two or more different indices from this set, the corresponding terms become negligible in the thermodynamic limit, as can be easily verified with the help of the P_σ -rule established in I. Explicitly, we find for the third case, i.e., (2.26),

$$\underbrace{\text{Tr}_{s+1, s+2}}_{\sim \Omega^2} \underbrace{P^{1 \dots s+2} U(1, 2, s+1, s+2) \cdots}_{\sim \Omega^{-3}} \sim \Omega^{-1} \quad (2.29)$$

Consequently, each cluster $U(I)$ must contain one and only one index from the set $\{1, \dots, s\}$ [as is the case in (2.25)], and it follows that

$$\begin{aligned}
 & P^{1 \dots s} \rho_{1 \dots s}(t) \\
 &= \sum_{\substack{r_1, \dots, r_s \\ 0}}^{\infty} \text{Tr}_{s+1 \dots s+r} P^{1 \dots s+r} U(1I_1) \dots U(sI_s) \rho_{1 \dots s+r}(t_0) \\
 &\quad \text{for } \Omega \rightarrow \infty
 \end{aligned} \tag{2.30}$$

with

$$I_i = \begin{cases} \{s + r_1 + \dots + r_{i-1} + 1, \dots, s + r_2 + \dots + r_i\} & \text{for } r_i \geq 1 \\ \emptyset & \text{otherwise} \end{cases} \tag{2.31}$$

and

$$r = \sum_{i=1}^s r_i \tag{2.32}$$

Next we decompose $\rho_{1 \dots k}(t_0)$, analogously to $e^{-iL t}$, into the following cluster series:

$$\rho_{1 \dots k}(t_0) = \sum_{l=1}^k \sum_{I_1, \dots, I_k} g(I_1) \dots g(I_k) \tag{2.33}$$

Making use of the fact that, according to Eq. (2.18), $g(I)$ also possesses the cluster property, it then follows with the same argument as above that in Eq. (2.30) the clusters $g(I)$, which connect any indices from $\{1, \dots, s\}$ [e.g., $g(12)$], or $U(1, s+1) U(2, s+2) g(s+1, s+2)$, can be omitted in the thermodynamic limit. Hence, Eq. (2.30) reduces to

$$\begin{aligned}
 & P^{1 \dots s} \rho_{1 \dots s}(t) \\
 &= \sum_{\substack{r_1, \dots, r_s \\ 0}}^{\infty} \text{Tr}_{s+1 \dots s+r} P^{1 \dots s+r} U(1I_1) \dots U(sI_s) \\
 &\quad \times \rho_{1I_1}(t_0) \dots \rho_{sI_s}(t_0) \\
 &= \prod_{i=1}^s P^i \sum_{r=0}^{\infty} \frac{N!}{(N-r-1)!} \text{Tr}_{N-i} U(is+1 \dots s+r) \rho(t_0) \pi \\
 &\quad \text{for } \Omega \rightarrow \infty
 \end{aligned} \tag{2.34}$$

where Eq. (2.15) has been used in the last step.

From Eqs. (2.22), (2.23), and (2.28) it now follows that

$$\begin{aligned}
 & P^{1 \cdots s} \rho_{1 \cdots s}(t) \\
 &= \prod_{i=1}^s P^i N \operatorname{Tr}_{N-i} \{ \exp \{ -i(t-t_0) [L(1 \cdots \dot{i} \cdots s) + L(is+1 \cdots N)] \} \} \\
 &\quad \times \rho(t_0) \pi \tag{2.35}
 \end{aligned}$$

Since in each factor of the last equation the indices $1, \dots, s$ are distinguished and therefore cannot produce N factors, we may replace the dynamical superoperators by $e^{-iL(t-t_0)}$ in the thermodynamic limit. Thus, with Eq. (2.15), we finally arrive at the desired result, the factorization theorem (2.19), which accomplishes the proof.

We note that, according to the above proof, Eq. (2.19) is still valid for the more general case, where the initial value $\rho_{1 \cdots s}(t_0)$ can be written as a cluster series of the form (2.33) [and not only as an (anti-) symmetrized product of one-particle operators as we have assumed in Eq. (2.18)].

Finally, we would like to emphasize the facts that formula (2.19) is valid for all times $t \geq t_0$ (with t_0 finite) and that this factorization theorem should not be confused with Bogoliubov's functional assumption,⁽²⁹⁾ according to which the higher-order reduced density operators are determined as explicitly time-independent functionals of $\rho_1(t)$ in the limit $t_0 \rightarrow -\infty$. The only limit necessary in the above proof is the thermodynamic limit, which allows us to omit corrections of the relative order Ω^{-1} (and smaller).

3. CLOSED EQUATION FOR $\rho_1(t)$

We now start the actual derivation of a generalized kinetic equation for a homogeneous system consisting of N identical interacting fermions or bosons.

In this section, as a first step toward this aim, we expand the resolvent $(\varepsilon + iL)^{-1}$ occurring in the Laplace transform of $\rho_1(t)$ into an explicit cluster series by a simple iteration procedure. Then, making use of the factorization theorem established in the foregoing section, we obtain a closed integrodifferential equation for $\rho_1(t)$, where only one-particle density operators $\rho_i(t)$ occur.

If we considered a system of distinguishable particles obeying classical, i.e., Boltzmann statistics, this closed equation would be almost the final result, from which kinetic equations, such as the Boltzmann or Choh-Uhlenbeck equations, could immediately be obtained. However, in the quantum-statistical case considered here, the action of the (anti-) symmetrizer π complicates the situation in that a nontrivial resummation is

necessary in order to obtain the quantum-statistical generalization, e.g., of the Boltzmann and the Choh-Uhlenbeck equations. This resummation procedure will be discussed in the next section.

To begin with, let us introduce the Laplace transform of $\rho_{1\dots s}(t)$,

$$\tilde{\rho}_{1\dots s}(\varepsilon) = \int_0^\infty dt e^{-\varepsilon t} \rho_{1\dots s}(t), \quad \varepsilon > 0 \tag{3.1}$$

Choosing the initial time $t_0 = 0$ in Eq. (2.4), we obtain explicitly

$$P^1 \tilde{\rho}_1(\varepsilon) = N \text{Tr}_{2\dots N} P \frac{1}{\varepsilon + iL} \rho(0) \pi \tag{3.2}$$

Then applying the identity

$$\frac{1}{X + Y} = \frac{1}{X} - \frac{1}{X} Y \frac{1}{X + Y} \tag{3.3}$$

to $[\varepsilon + i(L_0 + L_\nu)]^{-1}$, we find, with $PL_0 = 0$,

$$\varepsilon P^1 \tilde{\rho}_1(\varepsilon) - P^1 \rho_1(0) = A_1(\varepsilon) \tag{3.4}$$

with

$$A_1(\varepsilon) = -N(N-1) \text{Tr}_{2\dots N} P iL_{12} \frac{1}{\varepsilon + iL} \rho(0) \pi \tag{3.5}$$

where we have used that

$$\text{Tr}_{i\dots k} L(i \dots k) A = 0 \tag{3.6}$$

Our aim now is to obtain an explicit cluster series for the dynamical factor occurring in $A_1(\varepsilon)$. Before doing so, let us remark that we are ultimately interested in the small- ε behavior of $A_1(\varepsilon)$ (see Sections 5 and 6). However, as discussed in I and II, a naive cluster expansion of the resolvent $(\varepsilon + iL)^{-1}$ does not work in this case, since the resulting cluster series diverges term by term as $\varepsilon \rightarrow 0$ due to the occurrence of van Hove's diagonal singularities⁽²¹⁾ (for details we refer to I and II). As shown in II, this divergence difficulty can be removed by the help of the following symmetrized s -particle diagonal and nondiagonal projectors:

$$\bar{P}^{1\dots s} = \sum_{\sigma \in S_s} \pi_\sigma P^{1\dots s} \pi_\sigma^{-1}, \quad \bar{Q}^{1\dots s} = 1 - \bar{P}^{1\dots s} \tag{3.7}$$

π_σ is defined in Eq. (2.9) and π_σ^{-1} denotes its inverse. In matrix notation this reads

$$(\bar{P}^{1\dots s} A)_{kk'} = A_{kk'} \sum_{\sigma \in S_s} \delta_{k, \sigma(k^s)} = A_{kk'} \delta_{\{k^s\}, \{k^s\}} \tag{3.8}$$

Here $\sigma(k^{rs}) = (\mathbf{k}_{\sigma(1)}, \dots, \mathbf{k}_{\sigma(s)})$ and $\delta_{\{k^s\}, \{k^{rs}\}}$ equals 1 if the sets $\{k^s\} \equiv \{\mathbf{k}_1, \dots, \mathbf{k}_s\}$ and $\{k^{rs}\}$ are equal and 0 otherwise. The last equality sign in Eq. (3.88) holds only for $s \ll N$.⁽²³⁾

The reason for introducing these projectors is that in a term of the form $L_\nu[\varepsilon + iL(1 \dots s)]^{-1}$ (as it occurs in a cluster or perturbation expansion) the most divergent term as $\varepsilon \rightarrow 0$ can easily be extracted by the help of Eq. (3.3) and $L_0(1 \dots s) \bar{P}^{1 \dots s} = 0$:

$$L_\nu \frac{1}{\varepsilon + iL(1 \dots s)} = \frac{1}{\varepsilon} L_\nu \left[1 - \frac{1}{\varepsilon + iL(1 \dots s)} iL_\nu(1 \dots s) \right] \bar{P}^{1 \dots s} + L_\nu \frac{1}{\varepsilon + iL(1 \dots s)} \bar{Q}^{1 \dots s} \tag{3.9}$$

where, in a given perturbation order [with respect to $L_\nu(1 \dots s)$], the second term always possesses an ε^{-1} factor less than the first term due to the effect of $\bar{Q}^{1 \dots s}$.⁽²³⁾

After these remarks let us return to Eq. (3.5) and begin with the cluster expansion. The following iteration procedure is an alternative but equivalent method to the derivations given in I and II [see, e.g., (II.3.17)–(II.3.25)]. Its main feature is that only s -particle projectors $\bar{P}^{1 \dots s}$ and $\bar{Q}^{1 \dots s}$ will occur and that therefore the explicit representation of the N -particle version of $\bar{P}^{1 \dots s}$ is not needed. This might be advantageous especially if projectors are used (e.g., in inhomogeneous systems), which are more complicated than the diagonal projector $\bar{P}^{1 \dots s}$ considered in the present work.

Now, in a first step, by using Eq. (3.3) and $\bar{P}^{12} + \bar{Q}^{12} = 1$, we write identically

$$\frac{1}{\varepsilon + iL} = \frac{1}{\varepsilon + iL(12)} (\bar{P}^{12} + \bar{Q}^{12}) \left\{ 1 - i[L - L(12)] \frac{1}{\varepsilon + iL} \right\} \tag{3.10}$$

which, inserted into Eq. (3.5), leads to

$$A_1(\varepsilon) = -\frac{N!}{(N-2)!} \text{Tr}_{2 \dots N} PiL_{12} \frac{1}{\varepsilon + iL(12)} \bar{P}^{12} \left(1 - iL_\nu \frac{1}{\varepsilon + iL} \right) \rho(0)\pi - \frac{N!}{(N-2)!} \text{Tr}_{2 \dots N} PiL_{12} \frac{1}{\varepsilon + iL(12)} \bar{Q}^{12} \rho(0)\pi + \frac{N!}{(N-3)!} \text{Tr}_{2 \dots N} PiL_{12} \frac{1}{\varepsilon + iL(12)} \bar{Q}^{12} i(L_{13} + L_{23}) \frac{1}{\varepsilon + iL} \rho(0)\pi$$

for $\Omega \rightarrow \infty$ (3.11)

Here we have used Eq. (3.6) and the fact that $P^{3 \cdots N}[L - L(12)] = P^{3 \cdots N}(L_v - L_{12})$. Furthermore, we have omitted the term $L_{12}[\varepsilon + iL(12)]^{-1} \bar{P}^{12} L_{12}$, since it is of negligible order in Ω in the thermodynamic limit due to our P_q -rule.⁽¹²⁾

In a second step we replace the resolvent $(\varepsilon + iL)^{-1}$ occurring in the last term of the foregoing equation by

$$\frac{1}{\varepsilon + iL(123)} (\bar{P}^{123} + \bar{Q}^{123}) \left\{ 1 - i[L_v - L_v(123)] \frac{1}{\varepsilon + iL} \right\} \quad (3.12)$$

which, with Eq. (3.6) and the P_q -rule, reduces to

$$\begin{aligned} & \frac{1}{\varepsilon + iL(123)} \bar{P}^{123} \left[1 - iL_v \frac{1}{\varepsilon + iL} \right] + \frac{1}{\varepsilon + iL(123)} \bar{Q}^{123} \\ & - \frac{N-3}{\varepsilon + iL(123)} \bar{Q}^{123} i(L_{14} + L_{24} + L_{34}) \frac{1}{\varepsilon + iL} \end{aligned} \quad (3.13)$$

In a next step, one transforms the resolvent $(\varepsilon + iL)^{-1}$ occurring in the last term in (3.13) in a similar way. Iteration of this procedure yields the desired cluster series for the dynamical factor in A_1 :

$$\begin{aligned} A_1(\varepsilon) = & \sum_{s=2}^{N \rightarrow \infty} \text{Tr}_{2 \cdots s} P^{1 \cdots s} \bar{G}_{1 \cdots s}(\varepsilon) \bar{P}^{1 \cdots s} \bar{\rho}_{1 \cdots s}(\varepsilon) \\ & + \sum_{s=2}^{N \rightarrow \infty} \text{Tr}_{2 \cdots s} P^{1 \cdots s} \bar{G}_{1 \cdots s}(\varepsilon) \bar{Q}^{1 \cdots s} \rho_{1 \cdots s}(0), \quad \Omega \rightarrow \infty \end{aligned} \quad (3.14)$$

where the s -particle superoperator $\bar{G}_{1 \cdots s}$ is given by

$$\begin{aligned} \bar{G}_{1 \cdots s}(\varepsilon) = & (-1)^{s-1} iL_{12} \frac{1}{\varepsilon + iL(12)} \bar{Q}^{12} i(L_{13} + L_{23}) \frac{1}{\varepsilon + iL(123)} \\ & \times \cdots \frac{1}{\varepsilon + iL(1 \cdots s-1)} \bar{Q}^{1 \cdots s-1} i(L_{1s} + \cdots + L_{s-1s}) \frac{\varepsilon}{\varepsilon + iL(1 \cdots s)}, \\ & s \geq 2 \end{aligned} \quad (3.15)$$

We note that $\bar{G}_{1 \cdots s}$ is only essentially different from zero if the particles 1, ..., s are close together due to the assumed short-range nature of the interaction.^(12,23) Therefore, Eq. (3.14) indeed represents a cluster expansion of the dynamical factor. We point out, however, that, due to the action of the (anti-) symmetrizer π , Eq. (3.14) does not yet represent a cluster expansion of the whole expression, as will be shown in the next section.

We also note that a further decomposition of $\bar{G}_{1\dots s}$ with $\bar{P}^{1\dots k} + \bar{Q}^{1\dots k} = 1$ would not lead to new results, since the resulting $\bar{P}^{1\dots k}$ -contributions vanish in the thermodynamic limit due to the P_q -rule.⁽¹²⁾

To obtain finally a closed equation for $\rho_1(t)$, we assume now that $\rho_{1\dots s}(t)$ fulfills the initial condition (2.18). Thereby we exclude initial correlations due to the interaction; only statistical correlations due to FD or BE statistics are taken into account. Since, besides these statistical correlations, the situation here is very similar to the classical case, we refer the interested reader to ref. 4 for a physical interpretation of the initial condition assumed here. Since $\pi_\sigma \rho_{1\dots s}(t) = \rho_{1\dots s}(t)$ for all $\sigma \in S_s$, it then follows from the factorization theorem (2.19) that

$$\begin{aligned} \bar{P}^{1\dots s} \rho_{1\dots s}(t) &= \pi^{1\dots s} P^{1\dots s} \rho_{1\dots s}(t) \\ &= \pi^{1\dots s} \rho_1(t) \cdots \rho_s(t) \quad \text{for } \Omega \rightarrow \infty \end{aligned} \quad (3.16)$$

On the other hand, the inhomogeneous (i.e., second) term in Eq. (3.14) vanishes in this case, since then

$$\bar{Q}^{1\dots s} \rho_{1\dots s}(0) = 0 \quad \text{for } \Omega \rightarrow \infty \quad (3.17)$$

Thus, transforming Eq. (3.4) back to time space and making use of Eqs. (3.14)–(3.17), one eventually gets

$$\dot{\rho}_1(t) = \int_0^t dt' \sum_{s=2}^{\infty} \text{Tr}_{2\dots s} H_{1\dots s}(t') \rho_1(t-t') \cdots \rho_s(t-t') \quad \text{for } \Omega \rightarrow \infty \quad (3.18)$$

where the inverse Laplace transform $H_{1\dots s}(t)$ is given by

$$H_{1\dots s}(t) = \frac{1}{2\pi i} \int_{\gamma} d\varepsilon e^{\varepsilon t} P^{1\dots s} \bar{G}_{1\dots s}(\varepsilon) \pi^{1\dots s} P^{1\dots s} \quad (3.19)$$

The contour γ encircles the singularity $\varepsilon = 0$ that arises in $\bar{G}_{1\dots s}(\varepsilon)$.

Equation (3.18) is a closed non-Markovian integrodifferential equation for $\rho_1(t)$. It has been derived under the assumption that the initial condition (2.18) is fulfilled. We also note that this equation is formally exact in the thermodynamic limit and holds for all $t > 0$.

As already remarked, in the case with Boltzmann statistics (i.e., if π is replaced by 1) Eq. (3.18) reduces to a generalized kinetic equation, from which the Boltzmann equation (i.e., $s=2$) and the Choh-Uhlenbeck correction (i.e., $s=3$) could easily be obtained in the Markovian approximation. Since these equations are special cases of the quantum-statistical equations derived below, this shall not be worked out here.

4. RESUMMATION AND QUANTUM-STATISTICAL CLUSTER SERIES

We have already pointed out that the series occurring on the rhs of Eq. (3.18) does not yet represent a true cluster expansion. This means that

$$\text{Tr}_{2\dots s} H_{1\dots s}(t') \rho_1(t-t') \cdots \rho_s(t-t')$$

contains also contributions with less than s particles and not only s -particle clusters (as would be the case if π were replaced by 1). The ultimate reason for this is that in special cases the s -particle trace, $\text{Tr}_{2\dots s}$, can reduce to $\text{Tr}_{2\dots s'}$, with $2 \leq s' < s$, due to the identity

$$\text{Tr}_i \pi_{ij} = \text{Tr}_j \pi_{ij} = \eta \tag{4.1}$$

for the permutation operator π_{ij} defined in Eq. (2.8) (η equals 1 for bosons and -1 for fermions).

By making use of the foregoing identity we have shown in II how the two-particle contributions in \bar{G}_{12} , \bar{G}_{123} , etc., can be extracted and resummed exactly. As a result the exchange-modified t -matrix has been obtained. It is now possible to generalize this procedure, i.e., to extract the s -particle contributions in $\bar{G}_{1\dots s}$, $\bar{G}_{1\dots s+1}$, etc., and to resume them in a closed form. This then leads to a quantum-statistical renormalized cluster series where now the s -particle contributions are grouped together. The details of this resummation procedure are given in the Appendix. As result, we obtain the following exact generalized kinetic equation from Eq. (3.18):

$$\dot{\rho}_1(t) = \int_0^t dt' \sum_{s=2}^{\infty} \text{Tr}_{2\dots s} \hat{H}_{1\dots s}(t'; t-t') \rho_1(t-t') \cdots \rho_s(t-t') \quad \text{for } \Omega \rightarrow \infty \tag{4.2}$$

where

$$\hat{H}_{1\dots s}(\tau; t) = \frac{1}{2\pi i} \int_{\gamma} d\varepsilon e^{\varepsilon\tau} P^{1\dots s} \hat{G}_{1\dots s}(\varepsilon; t) P^{1\dots s} \tag{4.3}$$

Here, the new cluster superoperators $\hat{G}_{1\dots s}(\varepsilon; t)$ are now functionals of the one-particle distribution operators $\rho_1(t), \dots, \rho_s(t)$ and depend therefore also on the time t . Explicitly, one has (see Appendix)

$$\begin{aligned} \hat{G}_{1\dots s}(\varepsilon; t) = & (-1)^{s-1} i \bar{L}_{12} \frac{1}{\varepsilon + i\hat{L}(12; t)} iQ^{12} (\bar{L}_{13} + \bar{L}_{23} + \bar{L}_{12,3}) \\ & \times \frac{1}{\varepsilon + i\hat{L}(123; t)} \cdots iQ^{1\dots s-1} \\ & \times \left(\bar{L}_{1s} + \cdots + \bar{L}_{s-1s} + \sum_{1 \leq i < j \leq s-1} \bar{L}_{ij,s} \right) \\ & \times \frac{\varepsilon}{\varepsilon + i\hat{L}(1\dots s; t)} - (-1)^{s-1} M_s(\varepsilon; t) \end{aligned} \tag{4.4}$$

where the new Liouville operators are defined by

$$\bar{L}_{12}A = L_{12}(1 + \pi_{12})A \quad (4.5)$$

$$\bar{L}_{12,3}A = (\pi_{13} + \pi_{23})V_{12}A - AV_{12}(\pi_{13} + \pi_{23}) \quad (4.6)$$

$$\tilde{L}_{12}(t)A = \eta[\rho_1(t) + \rho_2(t)]V_{12}A - \eta AV_{12}[\rho_1(t) + \rho_2(t)] \quad (4.7)$$

$$\hat{L}_{12}(t) = L_{12} + \tilde{L}_{12}(t) \quad (4.8)$$

$$\hat{L}(1 \cdots s; t) = L_0(1 \cdots s) + \sum_{1 \leq i < j \leq s} \hat{L}_{ij}(t) \quad (4.9)$$

$M_s(\varepsilon; t)$ is defined as follows. Due to the identity

$$\text{Tr}_3 \bar{L}_{12,3} \rho_3(t) A_{12} = \tilde{L}_{12}(t) A_{12} \quad (4.10)$$

which follows from Eq. (4.1), there are still contributions containing less than three particles in the first term on the rhs of (4.4) [when inserted into Eq. (4.2)]. These contributions, denoted by $M_s(\varepsilon; t)$, must therefore be subtracted. For instance, one finds

$$M_2(\varepsilon; t) = 0 \quad (4.11)$$

$$M_3(\varepsilon; t) = i\bar{L}_{12} \frac{1}{\varepsilon + i\hat{L}(12; t)} iQ^{12} \bar{L}_{12,3} \frac{\varepsilon}{\varepsilon + i\hat{L}(12; t)} \quad (4.12)$$

etc. (Note that Q^{12} in M_3 can actually be omitted due to the P_q -rule.)

If one compares $\bar{G}_{1 \cdots s}(\varepsilon)$ [in particular, in the form given in the Appendix, (A.1)] with $\hat{G}_{1 \cdots s}(\varepsilon; t)$, one recognizes that both formulas have almost the same form [apart from $M_s(\varepsilon; t)$], only the resolvents $[\varepsilon + iL(1 \cdots s')]^{-1}$, $2 \leq s' \leq s$, have been replaced by $[\varepsilon + i\hat{L}(1 \cdots s'; t)]^{-1}$ as a result of the resummation. This quantum-statistical Liouville operator $\hat{L}(1 \cdots s; t)$ can also be expressed by a generalized Hamilton operator, which, however, is no longer self-adjoint:

$$\hat{L}(1 \cdots s; t)A = \hat{H}(1 \cdots s; t)A - A\hat{H}^+(1 \cdots s; t) \quad (4.13)$$

where

$$\hat{H}(1 \cdots s; t) = H_0(1 \cdots s) + \sum_{1 \leq i < j \leq s} S_{ij}(t) V_{ij} \quad (4.14)$$

with

$$S_{ij}(t) = 1 + \eta\rho_i(t) + \eta\rho_j(t) \quad (4.15)$$

It then follows that

$$e^{i\hat{L}(1\cdots s;t)}A = e^{i\hat{H}(1\cdots s;t)}Ae^{-i\hat{H}^+(1\cdots s;t)} \quad (4.16)$$

These relations now show that, e.g., in a perturbation expansion of the rhs of Eq. (4.16), the occupation probability of the intermediate free particle states is modified by the weighting operator $S_{ij}(t)$. This weighting, in turn, is determined by the FD or BE statistics. We note here that $S_{ij}(t)$ also occurs in other formalisms, e.g., in the systematic diagrammatic perturbation method of ref. 19 or in the semiphenomenological approach of ref. 16 [where also a further discussion of $S_{ij}(t)$ with further references can be found]. We might therefore consider $\hat{H}(1\cdots s;t)$ [$\hat{L}(1\cdots s;t)$] a renormalized s -particle Hamiltonian (Liouvillian), in which the influence of the other particles of the N -particle system via the statistics has been taken into account. In this sense, we might also say that the kinetic equation (4.2) together with (4.4) represents a renormalized integrodifferential equation for the one-particle distribution operator $\rho_1(t)$ of a homogeneous non-equilibrium system. This exact kinetic equation, being the main result of our general discussion, may now serve as starting point for the derivation of approximate kinetic equations, in particular, in the asymptotic time regime. It must be noted, however, that for $\varepsilon \rightarrow 0$ further divergences (besides the eliminated first divergences) occur in $\hat{G}_{1\dots s}(\varepsilon;t)$ for $s \geq 4$ (in three dimensions) due to the quantum-statistical analog of the well-known classical^(3,7) or semiclassical⁽¹²⁾ ring events. The question of whether these ring terms are the next leading divergences or whether there are equally or more singular terms (as one should expect from the discussion of the Lorentz gas⁽³⁰⁾) needs further investigation and will not be treated here. In any case, the generalized kinetic equation (4.2) together with (4.4) are given in a form which is well appropriate for a further analysis with the help of binary collision and scattering length expansions.

Finally, we remark that $\hat{G}_{1\dots s}$ can always be brought to a form in which the projector Q (P , resp.) no longer occurs. This is simply achieved by replacing all Q 's by $1 - P$ and by applying the P_q -rule to the resulting terms with P 's (see the example \hat{G}_{123} treated in Section 6). In this way, however, $\hat{G}_{1\dots s}$, becoming a sum of different terms, loses its compact and transparent form.

5. QUANTUM-STATISTICAL BOLTZMANN EQUATION

As a first application of the general formalism presented in the preceding sections, we now derive the quantum-statistical generalization of the homogeneous Boltzmann equation in the binary collision and Markovian approximations. Thereby we obtain a Uehling-Uhlenbeck⁽³¹⁾

type equation with a scattering cross section, which, due to exchange effects, is now time-dependent through its functional dependence on the one-particle distributions $\rho_1(t)$ and $\rho_2(t)$. To arrive at this result we can proceed in close analogy to the derivation of the classical Boltzmann equation from generalized kinetic equations (see, e.g., ref. 4).

In the binary collision approximation the generalized kinetic equation (4.2) reduces to

$$\dot{\rho}_1(t) = \int_0^t dt' \text{Tr}_2 \hat{H}_{12}(t'; t-t') \rho_1(t-t') \rho_2(t-t') \quad (5.1)$$

Thereby all collisions involving only two particles are taken into account, including the many-particle correlations arising from exchange effects.

Next we determine the asymptotic form of the foregoing equation, i.e., we consider times t large compared to a typical binary collision time τ_c . In this case, Eq. (5.1) can be further simplified by assuming, in analogy to the classical case, that the kernel $\hat{H}_{12}(\tau; t-t')$ decays rapidly to zero for times τ larger than τ_c . We may then extend the upper limit of the time integral in Eq. (5.1) to infinity with negligible (or at least small) error and expand $\hat{H}_{12}(\tau; t-t')$ and $\rho_i(t-t')$ around t as follows:

$$\hat{H}_{12}(\tau; t-t') = \hat{H}_{12}(\tau; t) - t' \frac{\partial}{\partial t} \hat{H}_{12}(\tau; t) + \dots \quad (5.2)$$

$$\rho_i(t-t') = \rho_i(t) - t' \dot{\rho}_i(t) + \dots \quad (5.3)$$

Inserting these expansions into Eq. (5.1), we obtain

$$\dot{\rho}_1(t) = \lim_{\varepsilon \rightarrow 0^+} \text{Tr}_2 P^{12} \hat{G}_{12}(\varepsilon; t) P^{12} \rho_1(t) \rho_2(t) + \sum_{k=1}^{\infty} \hat{R}_k(t) \quad (5.4)$$

The first term on the rhs represents the Markovian approximation of Eq. (5.1). The corrections $\hat{R}_k(t)$, coming from the higher order terms in the expansions (5.2) and (5.3), describe the time retardation effects due to the finite duration of a collision and are of the order $(\tau_c/t)^k$. In particular, one finds for $\hat{R}_1(t)$

$$\begin{aligned} \hat{R}_1(t) = & \lim_{\varepsilon \rightarrow 0^+} \text{Tr}_{23} \hat{G}'_{12}(\varepsilon; t) [\hat{G}_{13}(\varepsilon; t) + \hat{G}_{23}(\varepsilon; t)] \rho_1(t) \rho_2(t) \rho_3(t) \\ & + \lim_{\varepsilon \rightarrow 0^+} \text{Tr}_{234} M'_3(\varepsilon; t) \hat{G}_{34}(\varepsilon; t) \rho_1(t) \dots \rho_4(t) \end{aligned} \quad (5.5)$$

where the prime denotes the derivative with respect to ε . For brevity, the P 's, resulting automatically due to momentum conservation, have been omitted. It is obvious that \hat{R}_1 involves more than two particles [note that

the term with M'_3 actually involves only three particles (like the other terms) due to an identity of the type (4.10)]. Therefore, in the considered binary collision approximation the time retardation effects can be omitted. Finally, as shown in II (see Appendix D there), the first term on the rhs of Eq. (5.4) can be transformed identically to a Uehling-Uhlenbeck⁽³¹⁾ collision operator form with, however, an exchange-modified, time-dependent scattering cross section. Hence, we obtain from Eq. (5.4) the homogeneous quantum-statistical Boltzmann equation in the following form:

$$\dot{f}(\mathbf{k}_1; t) = \hat{I}_2(\mathbf{k}_1; t) \tag{5.6}$$

where the two-particle collision integral is given by

$$\begin{aligned} \hat{I}_2(\mathbf{k}_1; t) = & 4\pi \sum_{\mathbf{k}_2, \bar{\mathbf{k}}_1, \bar{\mathbf{k}}_2} |\langle \mathbf{k}_1 \mathbf{k}_2 | \hat{t}_{12}^+(\varepsilon_{12}; t) \frac{1}{2}(1 + \pi_{12}) | \bar{\mathbf{k}}_1 \bar{\mathbf{k}}_2 \rangle|^2 \\ & \times \delta(\varepsilon_{12} - \bar{\varepsilon}_{12}) \{ \bar{f}_1 \bar{f}_2 (1 + \eta f_1) (1 + \eta f_2) \\ & - (1 + \eta \bar{f}_1) (1 + \eta \bar{f}_2) f_1 f_2 \} \end{aligned} \tag{5.7}$$

with

$$\begin{aligned} \bar{f}_1 = f(\bar{\mathbf{k}}_1; t) &= \langle \bar{\mathbf{k}}_1 | \rho_1(t) | \bar{\mathbf{k}}_1 \rangle \\ \bar{\varepsilon}_{12} &= \frac{1}{2m} (\bar{\mathbf{k}}_1^2 + \bar{\mathbf{k}}_2^2) \end{aligned} \tag{5.8}$$

etc. The exchange modified t -matrix, defined by

$$\hat{t}_{12}^+(E; t) = \lim_{\varepsilon \rightarrow 0^+} V_{12} \frac{1}{\varepsilon + i[\hat{H}(12; t) - E]} \{ \varepsilon + i[H_0(12) - E] \} \tag{5.9}$$

is, as stated above, now time dependent through its dependence on the distribution operators $\rho_1(t)$ and $\rho_2(t)$ occurring in $\hat{H}(12; t)$ [see Eq. (4.14)]. Therefore, the collision integral \hat{I}_2 is a complicated functional of $\rho_1(t)$ and $\rho_2(t)$ and is not only trilinear in the f 's (the quartic terms cancel) as is the case in the original Uehling-Uhlenbeck equation or in the Born approximation of Eq. (5.6). Nevertheless, the equilibrium solution of Eq. (5.6) is still given by the Fermi-Dirac or Bose-Einstein distribution, as is easily checked.

Finally, we remark that the above form of the Boltzmann equation (with the same scattering cross section) was first obtained by Boercker and Dufty⁽¹⁶⁾ in a semiphenomenological derivation which is based on a closure approximation of the quantum BBGKY hierarchy⁽³²⁾ for reduced distribution operators. For a discussion of the exchange-modified t -matrix we refer to this work. See also ref. 17 of the same authors, where an *ab initio* deriva-

tion of the linearized version of the above quantum-statistical Boltzmann equation is given, which is based on a formal closure of the BBGKY hierarchy and on cluster expansions.

The same⁽³³⁾ or similar⁽³⁴⁾ results as obtained in this section have been derived by other authors, too, using the Kadanoff–Baym hierarchy of equations of motion for nonequilibrium real-time Green’s functions.⁽³⁵⁾

For investigations restricted to second-order approximations (in the interaction) of the collision operator see, e.g., refs. 13, 14, and 36.

6. QUANTUM-STATISTICAL CHO–UHLENBECK EQUATION

To go beyond a Boltzmann equation description, higher order clusters involving more than two particles must be taken into account. In particular, the first correction to the two-particle collision integral \hat{I}_2 occurring in the Boltzmann equation (5.6) is given by the three-particle clusters \hat{G}_{123} and \hat{R}_1 (the latter describes the first time-retardation correction). As will be shown below, \hat{G}_{123} together with \hat{R}_1 leads to the quantum-statistical generalization of the (semi-) classical Choh–Uhlenbeck collision operator.

Therefore, considering the generalized kinetic equation (4.2) in the triple collision and Markovian approximation, thereby retaining the first time-retardation correction (the higher order corrections involve more than three particles), we arrive at the quantum-statistical generalization of the (semi-) classical Choh–Uhlenbeck equation

$$\dot{f}(\mathbf{k}_1; t) = \hat{I}_2(\mathbf{k}_1; t) + \hat{I}_3(\mathbf{k}_1; t) \quad (6.1)$$

\hat{I}_2 is given in Eq. (5.7). The three-particle collision integral is defined by

$$\hat{I}_3(\mathbf{k}_1; t) = \lim_{\varepsilon \rightarrow 0^+} [\text{Tr}_{23} \hat{G}_{123}(\varepsilon; t) \rho_1(t) \rho_2(t) \rho_3(t)]_{\mathbf{k}_1, \mathbf{k}_1} + \hat{R}_1(\mathbf{k}_1; t) \quad (6.2)$$

with $\hat{R}_1(\mathbf{k}_1; t) = (\hat{R}_1(t))_{\mathbf{k}_1, \mathbf{k}_1}$ [see Eq. (5.5)]. In deriving this result, we have used similar arguments and assumptions with respect to the Markovian limit as in Section 5.

As already pointed out in the general discussion of Section 4, the main feature of \hat{I}_3 (as well as of \hat{I}_2) is its renormalized form brought about by the quantum statistics. That means the effect of the other particles via the statistics on the three-particle cluster is already incorporated in \hat{G}_{123} through the renormalized Liouville operators $\hat{L}(12; t)$ and $\hat{L}(123; t)$. Due to this many-body effect the collision integral \hat{I}_3 becomes a complicated functional of the one-particle distribution operator $\rho_1(t)$, whereas in the (semi-) classical case \hat{I}_3 is only of third order in $\rho_1(t)$ (see below).

To make the close connection between \hat{I}_3 and its semiclassical counter-

part (i.e., the Choh-Uhlenbeck collision operator) more transparent, we shall now rewrite \hat{I}_3 in an alternative form. For that purpose we have to eliminate the nondiagonal projector Q^{12} occurring in \hat{G}_{123} . Hence, replacing there Q^{12} by $1 - P^{12}$ and applying the P_q -rule to the resulting terms with P^{12} , we immediately find

$$P\hat{G}_{123}(\varepsilon; t)P = P \left\{ \hat{G}_{123}^0(\varepsilon; t) - \frac{1}{\varepsilon} \hat{G}_{12}(\varepsilon; t) [\hat{G}_{13}(\varepsilon; t) + \hat{G}_{23}(\varepsilon; t)] \right\} P$$

for $\Omega \rightarrow \infty$ (6.3)

where \hat{G}_{123}^0 is equal to \hat{G}_{123} but without Q 's. In deriving this result, we have also used the momentum conservation due to which one has, e.g., $P\hat{G}_{12}P^{12}\hat{G}_{13}P = P\hat{G}_{12}\hat{G}_{13}P$ (see Appendix B of I). This \hat{G}_{123}^0 contains now first divergences as $\varepsilon \rightarrow 0$, coming from successive isolated binary collisions (which are excluded in \hat{G}_{123} due to the action of Q). These singular terms, however, are exactly compensated by the second term (being proportional to ε^{-1}) in the above equation.

Inserting Eq. (6.3) into \hat{I}_3 , we obtain

$$\hat{I}_3(\mathbf{k}_1, t) = \hat{I}_3^{(1)}(\mathbf{k}_1; t) + \hat{I}_3^{(2)}(\mathbf{k}_1; t) \tag{6.4}$$

where $\hat{I}_3^{(i)}(\mathbf{k}_1; t) = \langle \mathbf{k}_1 | \hat{I}_3^{(i)}(t) | \mathbf{k}_1 \rangle$, with

$$\begin{aligned} \hat{I}_3^{(1)} &= - \int_0^\infty d\tau \text{Tr}_{23} \bar{L}_{12} \hat{S}_{-\tau}(12; t) \{ (\bar{L}_{13} + \bar{L}_{23} + \bar{L}_{12,3}) \hat{S}_{-\infty}(123; t) \\ &\quad - \hat{S}_{-\infty}(12; t) [\bar{L}_{13} \hat{S}_{-\infty}(13; t) + \bar{L}_{23} \hat{S}_{-\infty}(23; t)] \} \rho_1(t) \rho_2(t) \rho_3(t) \\ &= - \int_0^\infty d\tau \text{Tr}_{23} L_{12} \hat{S}_{-\tau}(12; t) \{ (L_{13} + L_{23}) \hat{S}_{-\infty}(123; t) \pi^{123} \\ &\quad - \hat{S}_{-\infty}(12; t) [L_{13} \hat{S}_{-\infty}(13; t) \pi^{13} + L_{23} \hat{S}_{-\infty}(23; t) \pi^{23}] \pi^{12} \} \\ &\quad \times \rho_1(t) \rho_2(t) \rho_3(t) \end{aligned} \tag{6.5}$$

and

$$\hat{I}_3^{(2)}(t) = \lim_{\varepsilon \rightarrow 0^+} \text{Tr}_{23} \{ M'_3(\varepsilon; t) \text{Tr}_4 \hat{G}_{34}(\varepsilon; t) \rho_4(t) - M_3(\varepsilon; t) \} \rho_1(t) \rho_2(t) \rho_3(t) \tag{6.6}$$

The renormalized "streaming" superoperator is defined by

$$\begin{aligned} \hat{S}_{-\tau}(1 \dots s; t) A &\equiv e^{-i\tau \hat{L}(1 \dots s; t)} A \\ &= e^{-i\tau \hat{H}(1 \dots s; t)} A e^{i\tau \hat{H}^+(1 \dots s; t)} \end{aligned} \tag{6.7}$$

For the second equality in (6.5) we have made use of the identity

$$\bar{L}_{13} + \bar{L}_{23} + \bar{L}_{12,3} = (L_{13} + L_{23})(1 + \pi_{13} + \pi_{23})$$

(see Appendix B of II); we recall that $\pi^{123} = (1 + \pi_{12})(1 + \pi_{13} + \pi_{23})$ and $\pi^{ij} = (1 + \pi_{ij})$. We have written $\hat{I}_3^{(1)}$ in time space to exhibit its striking similarity with a form of the Choh-Uhlenbeck collision operator found in classical kinetic theory [see, e.g., Eq. (17.1) of ref. 37]. $\hat{I}_3^{(2)}$ is of purely quantum-statistical nature, i.e., it vanishes if exchange effects are neglected. The first term in Eq. (6.6) results from time-retardation effects, whereas the second one compensates the two-particle contributions occurring in the term with $\hat{S}_{-\infty}(123; t)$ in Eq. (6.5).

Let us now consider the semiclassical limit more explicitly, i.e., the case where the dynamics is still described by quantum mechanics, but where the particles obey classical Boltzmann statistics. Since this case has been studied by Résibois in detail within the Brussels formalism, we shall be very brief. First we note that in this limit $\hat{G}_{1\dots s}^0(\varepsilon; t)$ [$\hat{G}_{1\dots s}^0(\varepsilon; t)$, resp.] obviously reduces to $G_{1\dots s}(\varepsilon)$ [$G_{1\dots s}^0(\varepsilon)$, resp.], where $G_{1\dots s}(\varepsilon)$ is equal to $\bar{G}_{1\dots s}(\varepsilon)$ [see Eq. (3.15)], but with Q 's in place of \bar{Q} 's. Therefore, the semiclassical limit of \hat{I}_3 , denoted by I_3 , becomes ($\hat{I}_3^{(2)}$ vanishes)

$$I_3(t) = \lim_{\varepsilon \rightarrow 0^+} \text{Tr}_{23} \left\{ G_{123}^0(\varepsilon) + \left[G'_{12}(\varepsilon) - \frac{1}{\varepsilon} G_{12}(\varepsilon) \right] [G_{13}(\varepsilon) + G_{23}(\varepsilon)] \right\} \\ \times \rho_1(t) \rho_2(t) \rho_3(t) \quad (6.8)$$

which agrees with the expression obtained by Résibois.⁽²⁵⁾ As shown by Résibois,^{(25,26),3} Eq. (6.8) can be further transformed, with the result

$$I_3(t) = - \lim_{\tau \rightarrow \infty} \text{Tr}_{23} iL_{12} \{ S_{-\tau}(123) - S_{-\tau}(12) [S_{-\tau}(13) + S_{-\tau}(23)] \\ + S_{-\tau}(12) \} \rho_1(t) \rho_2(t) \rho_3(t) \quad (6.9)$$

where $S_{-\tau}(1\dots s) = e^{-i\tau L(1\dots s)}$. The rhs of the foregoing equation represents the semiclassical extension of the classical Choh-Uhlenbeck collision operator.⁽³¹⁾ Therefore, the kinetic equation (6.1) indeed represents the quantum-statistical generalization of the Choh-Uhlenbeck equation. For a further discussion of I_3 , we refer to the literature.^(25-27,38) Let us only note here that Résibois⁽²⁵⁾ has shown that, when bound states are excluded, I_3 cannot be expressed in terms of a probability transition $|\langle i | t(E_i) | f \rangle|^2 2\pi\delta(E_i - E_f)$ between an initial state i with energy E_i and a final state f (energy E_f), since this probability is diverging when there are

³ Note that on the rhs of Eq. (9) in ref. 26 a term $i\psi^{-(12)}(0)$ is missing.

more than two incident free particles. As a consequence, I_3 , as well as \hat{I}_3 , cannot be cast into a Boltzmann-like collision operator form (for the classical case see also ref. 39). This seems to be in conflict with ref. 34, where a quantum triple collision integral in terms of a probability transition of the above form is given, a result which seems not to be obtainable in the present formalism, at least not in a consistent triple collision approximation considered here.

Finally, let us note again (see also section 4) that, in the asymptotic time regime, it is not possible to write down kinetic equations including s -particle collision terms where $s > 3$, since in the Markovian limit these higher order collision integrals diverge due to the quantum-statistical analog of the ring events. Therefore, if one is interested in transport equations involving more than three-particle processes, it is necessary to perform further partial resummations in the renormalized cluster series on the rhs of the generalized kinetic equation (4.2). Only then one can hope to get better defined expressions in the long-time limit. Such a procedure, for instance, is very likely to be carried out if one tries to find quantum-statistical generalizations of the classical repeated ring equations^(9,40) in which many-body dynamic correlations are incorporated. We believe that the present formalism, especially the renormalized kinetic equation (4.2), is a suitable starting point for investigations directed at this aim.

APPENDIX

In this Appendix we derive Eq. (4.2) together with (4.4), starting from Eq. (3.18). In II [see (II.3.46)] we have shown that $\bar{G}_{1\dots s}(\varepsilon)\pi^{1\dots s}$ given in (3.15) can be rewritten as [replace i by $-i$ in (II.3.46)]

$$\begin{aligned} & \bar{G}_{1\dots s}(\varepsilon)\pi^{1\dots s} \\ &= (-1)^{s-1} i\bar{L}_{12} \frac{1}{\varepsilon + iL(12)} iQ^{12}(\bar{L}_{13} + \bar{L}_{23} + \bar{L}_{12,3}) \frac{1}{\varepsilon + iL(123)} \\ & \quad \times iQ^{123}(\bar{L}_{14} + \bar{L}_{24} + \bar{L}_{34} + \bar{L}_{12,4} + \bar{L}_{13,4} + \bar{L}_{23,4}) \frac{1}{\varepsilon + iL(1\dots 4)} \\ & \quad \times \dots \frac{1}{\varepsilon + iL(1\dots s-1)} \\ & \quad \times iQ^{1\dots s-1} \left(\bar{L}_{1s} + \dots + \bar{L}_{s-1,s} + \sum_{1 \leq i < j \leq s-1} \bar{L}_{ij,s} \right) \frac{\varepsilon}{\varepsilon + iL(1\dots s)} \end{aligned} \tag{A.1}$$

when applied to a symmetric operator A . The \bar{L}_{12} , $\bar{L}_{12,3}$, etc., are defined in Eqs. (4.5) and (4.6). Note that in (A.1) only the unsymmetrized projec-

tors $Q^{1\cdots s'}$ occur. The objective now is to collect all k -particle contributions occurring in

$$\sum_{s=2}^{\infty} \text{Tr}_{2\dots s} P^{1\cdots s} \bar{G}_{1\dots s}(\varepsilon) \pi^{1\cdots s} P^{1\cdots s} \rho_1(\bar{t}) \cdots \rho_s(\bar{t}), \quad (\bar{t} \equiv t - t') \quad (\text{A.2})$$

and to resum them. (For brevity we suppress in the following the time arguments.) The first step of this procedure has been discussed in Section 4 of II. For the sake of completeness and for notational reasons, however, this step is treated here again.

To find all binary collision contributions in (A.2), we note that, according to Eq. (4.10), the superoperators $\bar{L}_{12,l}$ can be reduced to a two-particle term which contains only V_{12} . In the first step, we therefore retain in (A.2) all L_{12} , \bar{L}_{12} , and $\bar{L}_{12,l}$ ($3 \leq l \leq s$) and the L_0 's, which reduce then to $L_0(12)$, since $L_0(12 \cdots s') P^{3\cdots s} = L_0(12) P^{3\cdots s}$ ($2 \leq s' \leq s$). We then obtain

$$\begin{aligned} & \sum_{s=2}^{\infty} \text{Tr}_{2\dots s} (-1)^{s-1} i \bar{L}_{12} \frac{1}{\varepsilon + iL(12)} iQ^{12} \bar{L}_{12,3} \frac{1}{\varepsilon + iL(12)} \\ & \quad \times iQ^{12} \bar{L}_{12,4} \frac{1}{\varepsilon + iL(12)} \cdots \bar{L}_{12,s} \frac{\varepsilon}{\varepsilon + iL(12)} P^{1\cdots s} \rho_1 \cdots \rho_s \\ & = - \sum_{s=2}^{\infty} \varepsilon \text{Tr}_2 P^{12} i \bar{L}_{12} \frac{1}{\varepsilon + iL(12)} \left(i \bar{L}_{12} \frac{-1}{\varepsilon + iL(12)} \right)^{s-2} P^{12} \rho_1 \rho_2 \\ & = - \varepsilon \text{Tr}_2 P^{12} i \bar{L}_{12} \frac{1}{\varepsilon + iL(12)} \frac{1}{1 + i \bar{L}_{12} \{1/[\varepsilon + iL(12)]\}} P^{12} \rho_1 \rho_2 \\ & = - \varepsilon \text{Tr}_2 P^{12} i \bar{L}_{12} \frac{1}{\varepsilon + i \bar{L}(12)} P^{12} \rho_1 \rho_2 \end{aligned} \quad (\text{A.3})$$

where we have used Eqs. (4.8)–(4.10), the fact that $Q^{1\cdots s'} P^{3\cdots s} = Q^{12} P^{3\cdots s}$, and the P_q -rule, due to which the Q 's can be omitted (the P^{12} part in $Q^{12} = 1 - P^{12}$ leads to a vanishing contribution for $\Omega \rightarrow \infty$). Then (A.3) gives the $s=2$ term in Eq. (4.2).

Analogously, in a next step, we collect all triple-collision terms in (A.2), by considering the particles 1, 2, and l ($2 < l \leq s$) and by retaining all L_{12} , L_{1l} , L_{2l} , \bar{L}_{12} , \bar{L}_{1l} , \bar{L}_{2l} , $\bar{L}_{12,j}$ ($2 < j \leq s$), $\bar{L}_{1l,k}$, and $\bar{L}_{2l,k}$ ($l < k \leq s$). This yields

$$\begin{aligned} & \sum_{s=2}^{\infty} \sum_{l=3}^s \text{Tr}_{2\dots s} P^{1\cdots s} (-1)^{s-1} i \bar{L}_{12} \frac{1}{\varepsilon + iL(12)} iQ^{12} \bar{L}_{12,3} \frac{1}{\varepsilon + iL(12)} \\ & \quad \times \cdots iQ^{12} \bar{L}_{12,l-1} \frac{1}{\varepsilon + iL(12)} iQ^{12} (\bar{L}_{1l} + \bar{L}_{2l} + \bar{L}_{12,l}) \frac{1}{\varepsilon + iL(12l)} iQ^{12l} \\ & \quad \times (\bar{L}_{12,l+1} + \bar{L}_{1l,l+1} + \bar{L}_{2l,l+1}) \frac{1}{\varepsilon + iL(12l)} \\ & \quad \times \cdots iQ^{12l} (\bar{L}_{12,s} + \bar{L}_{1l,s} + \bar{L}_{2l,s}) \frac{\varepsilon}{\varepsilon + iL(12l)} P^{1\cdots s} \rho_1 \cdots \rho_s \end{aligned}$$

$$\begin{aligned}
 &= \varepsilon \sum_{s=2}^{\infty} \sum_{l=3}^s \text{Tr}_{2l} P^{12l} (-1)^{s-1} i\bar{L}_{12} \frac{1}{\varepsilon + iL(12)} \\
 &\quad \times \left(iQ^{12} \tilde{L}_{12} \frac{1}{\varepsilon + iL(12)} \right)^{l-3} iQ^{12} (\bar{L}_{1l} + \bar{L}_{2l} + \bar{L}_{12,l}) \frac{1}{\varepsilon + iL(12l)} \\
 &\quad \times \left[iQ^{12l} (\tilde{L}_{12} + \tilde{L}_{1l} + \tilde{L}_{2l}) \frac{1}{\varepsilon + iL(12l)} \right]^{s-l} P^{12l} \rho_1 \rho_2 \rho_l \\
 &= (-1)^2 \varepsilon \sum_{l=3}^{\infty} \sum_{s=l}^{\infty} \text{Tr}_{23} P^{123} i\bar{L}_{12} \frac{1}{\varepsilon + iL(12)} \left(i\tilde{L}_{12} \frac{-1}{\varepsilon + iL(12)} \right)^{l-3} \\
 &\quad \times iQ^{12} (\bar{L}_{13} + \bar{L}_{23} + \bar{L}_{12,3}) \frac{1}{\varepsilon + iL(123)} \\
 &\quad \times \left[i(\tilde{L}_{12} + \tilde{L}_{13} + \tilde{L}_{23}) \frac{-1}{\varepsilon + iL(123)} \right]^{s-l} P^{123} \rho_1 \rho_2 \rho_3 \\
 &= (-1)^2 \varepsilon \text{Tr}_{23} P^{123} i\bar{L}_{12} \frac{1}{\varepsilon + iL(12)} \frac{1}{1 + i\tilde{L}_{12} \{1/[\varepsilon + iL(12)]\}} \\
 &\quad \times iQ^{12} (\bar{L}_{13} + \bar{L}_{23} + \bar{L}_{12,3}) \frac{1}{\varepsilon + iL(123)} \\
 &\quad \times \frac{1}{1 + i(\tilde{L}_{12} + \tilde{L}_{13} + \tilde{L}_{23}) \{1/[\varepsilon + iL(123)]\}} P^{123} \rho_1 \rho_2 \rho_3 \\
 &= (-1)^2 \varepsilon \text{Tr}_{23} P^{123} i\bar{L}_{12} \frac{1}{\varepsilon + i\tilde{L}(12)} iQ^{12} (\bar{L}_{13} + \bar{L}_{23} + \bar{L}_{12,3}) \\
 &\quad \times \frac{1}{\varepsilon + i\tilde{L}(123)} P^{123} \rho_1 \rho_2 \rho_3 \tag{A.4}
 \end{aligned}$$

In order not to overcount the two-particle contributions, which have already been taken into account in (A.3), we must subtract the term $\text{Tr}_{23} P^{123} M_3(\varepsilon; t) P^{123} \rho_1 \rho_2 \rho_3$ [see Eq. (4.12)] from (A.4), since this term contains only the particles 1 and 2 due to Eq. (4.10). This then leads to the $s=3$ term in Eq. (4.2).

It is now clear that one can proceed for the 4-, 5-, ..., particle contributions in exactly the same way, thereby arriving at Eq. (4.2).

Finally, we remark that, by reversing the steps leading from (3.15) to (A.1), $\hat{G}_{1\dots s}$ given in Eq. (4.4) can also be written as

$$\begin{aligned}
 \hat{G}_{1\dots s}(\varepsilon; t) &= (-1)^{s-1} iL_{12} \frac{1}{\varepsilon + i\tilde{L}(12; t)} i\bar{Q}^{12} (L_{13} + L_{23}) \frac{1}{\varepsilon + i\tilde{L}(123; t)} \\
 &\quad \times \dots \frac{1}{\varepsilon + i\tilde{L}(1 \dots s-1; t)} i\bar{Q}^{1\dots s-1} (L_{1s} + \dots + L_{s-1s}) \\
 &\quad \times \frac{\varepsilon}{\varepsilon + i\tilde{L}(1 \dots s; t)} \pi^{1\dots s} - (-1)^{s-1} M_s(\varepsilon; t) \tag{A.5}
 \end{aligned}$$

By comparing this form with (3.15), the effect of the renormalization procedure becomes evident (see also Section 4).

ACKNOWLEDGMENTS

We are grateful to Prof. A. Thellung for a critical reading of the manuscript and for helpful discussions. This work was supported by the Schweizerischer Nationalfonds.

REFERENCES

1. E. G. D. Cohen, in *Fundamental Problems in Statistical Mechanics*, Vol. II, E. G. D. Cohen, ed. (North-Holland, Amsterdam, 1968), pp. 228–275.
2. M. H. Ernst, L. K. Haines, and J. R. Dorfman, *Rev. Mod. Phys.* **41**:296 (1969).
3. J. R. Dorfman and H. van Beijeren, in *Statistical Mechanics*, B. J. Berne, ed. (Plenum Press, New York, 1977), Part B, p. 65.
4. P. Résibois and M. De Leener, *Classical Kinetic Theory of Fluids* (Wiley, New York, 1977).
5. J. R. Dorfman and E. G. D. Cohen, *Phys. Rev. A* **6**:776 (1972); *Phys. Rev. A* **12**:292 (1975).
6. L. E. Reichl, *A Modern Course in Statistical Physics* (University of Texas Press, Austin, Texas, 1980).
7. K. Kawasaki and I. Oppenheim, *Phys. Rev.* **139**:A1763 (1965).
8. Y. Kan and J. R. Dorfman, *Phys. Rev. A* **16**:2447 (1977).
9. M. H. Ernst and J. R. Dorfman, *Physica* **61**:157 (1972).
10. J. W. Dufty and J. A. McLennan, *Phys. Rev. A* **9**:1266 (1974).
11. J. R. Dorfman, *Physica A* **106**:77 (1981).
12. D. Loss and H. Schoeller, *Physica A* **150**:199 (1988).
13. R. Balescu, *Equilibrium and Nonequilibrium Statistical Mechanics* (Wiley, New York, 1975).
14. C. D. Boley and J. Smith, *Phys. Rev. A* **12**:661 (1975).
15. O. T. Valls, G. F. Mazenko, and H. Gould, *Phys. Rev. B* **19**:263 (1978).
16. D. B. Boercker and J. W. Dufty, *Ann. Phys.* **119**:43 (1979).
17. D. B. Boercker and J. W. Dufty, *Phys. Rev. A* **23**:1952 (1981).
18. R. Der and R. Haberlandt, *Physica A* **79**:597 (1975); *A* **86**:25 (1977).
19. R. Der, *Ann. Phys.* **34**:298 (1977).
20. J. W. Dufty, *Phys. Rev. A* **5**:2247 (1972).
21. L. van Hove, *Physica* **21**:517 (1955).
22. D. Loss, *Physica A* **139**:505 (1986).
23. D. Loss and H. Schoeller, *J. Stat. Phys.* **54**:765 (1989).
24. S. Choh and T. Uhlenbeck, *The Kinetic Theory of Dense Gases* (University of Michigan Report, 1958); M. Green, *Physica* **24**:393 (1958); E. G. D. Cohen, *Physica* **27**:163 (1961).
25. P. Résibois, *Physica* **31**:645 (1965).
26. P. Résibois, *Phys. Lett.* **9**:139 (1964).
27. P. Résibois, *Physica* **32**:1473 (1966).
28. P. Clavin, *C. R. Acad. Sci. (Paris) A* **274**:1022, 1085 (1972).
29. N. N. Bogoliubov, in *Studies in Statistical Mechanics*, Vol. I, G. E. Uhlenbeck and J. de Boer, eds. (North-Holland, Amsterdam, 1962).

30. T. R. Kirkpatrick and J. R. Dorfman, *J. Stat. Phys.* **30**:67 (1983); *Phys. Rev. A* **28**:1022 (1983).
31. E. A. Uehling and G. E. Uhlenbeck, *Phys. Rev.* **43**:552 (1933).
32. N. N. Bogoliubov, *Lectures on Quantum Statistics*, Vol. I (Gordon and Breach, New York, 1967).
33. P. Danielewicz, *Ann. Phys. (N.Y.)* **152**:239 (1984).
34. D. Kremp, M. Schlages, and T. Bornath, *J. Stat. Phys.* **41**:661 (1985).
35. L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1962).
36. P. Vasilopoulos and C. M. van Vliet, *Physica A* **121**:617 (1983).
37. Y. L. Klimontovich, *Kinetic Theory of Nonideal Gases and Nonideal Plasmas* (Pergamon Press, New York, 1982).
38. J. A. McLennan, *J. Stat. Phys.* **28**:257, 521 (1982).
39. M. S. Green, *Phys. Rev.* **136**:A905 (1964).
40. H. van Beijeren and M. H. Ernst, *J. Stat. Phys.* **21**:125 (1979).
41. E. Fick and G. Sauermann, *Quantenstatistik Dynamischer Prozesse*, Vols. I and IIa (Harry Deutsch, Thun-Frankfurt a.M., 1983, 1986).